Write your name here

Surname MODEL SOLUTIONS	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Further Pure Mathematics 1

Sample Assessment Material for first teaching September 2017

Time: 50 minutes

Paper Reference **8FM0/2B**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 5 questions in this question paper. The total mark for this paper is 40.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

a) $\sec x - \tan x = \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$\sin x = \frac{2t}{1+t^2}$ $\quad\cos x = \frac{1-t^2}{1+t^2}$

$\therefore \sec x - \tan x = \frac{1+t^2}{1-t^2} - \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}}$

$= \frac{1+t^2}{1-t^2} - \frac{2t}{1-t^2}$

$= \frac{1-2t+t^2}{1-t^2}$

$= \frac{(t-1)^2}{1-t^2}$

$= \frac{(1-t)^2}{(1+t)(1-t)}$

$= \frac{1-t}{1+t}$

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Question 1 continued

b) $$\frac{1-\sin x}{1+\sin x} = \frac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}$$

$$= \frac{1+t^2-2t}{1+t^2+2t}$$

$$= \frac{(t-1)^2}{(t+1)^2}$$

$$= \frac{(1-t)^2}{(1+t)^2}$$

$$= \left[\frac{1-t}{1+t}\right]^2$$

$$= \left[\sec x - \tan x\right]^2$$

(Total for Question 1 is 6 marks)

2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{dV}{dt} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)

$t_0 = 1, \; V_0 = 3$

we're estimating the value of the stock 30min after purchase (ie when $t = 1.5$)

$\frac{1.5-1}{2} = \frac{1}{4} = h$

$V_1 \doteq V_0 + h\left(\frac{dV}{dt}\right)_0 \rightarrow \frac{dV}{dt}_0 \doteq \frac{(3)^2 - 1}{1^2 + 3(1)} = 2$

$V_1 \doteq 3 + \frac{1}{4}(2) = 3.5$

$\left(\frac{dV}{dt}\right)_1 \doteq \frac{(3.5)^2 - (1.25)}{(1.25)^2 + (1.25)(3.5)} = \frac{176}{95}$

$V_2 \doteq 3.5 + \frac{1}{4}\left(\frac{176}{95}\right) \doteq 3.96$

so $\boxed{£\,396}$ will be our estimate

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Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of x for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

$$\frac{1}{x} < \frac{x}{x+2}$$

$\times\, x^2$

$$x < \frac{x^3}{x+2}$$

$\times\, (x+2)^2$

$$x(x+2)^2 < x^3(x+2)$$

$$x(x+2)[x+2-x^2] < 0$$

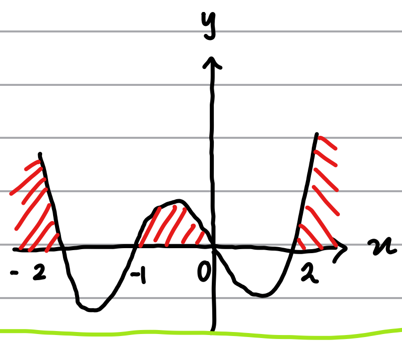
$$x(x+2)(-x^2+x+2) < 0$$

$$x(x+2)(x^2-x-2) > 0$$

$$x(x+2)(x-2)(x+1) > 0$$

+ve x^4 shape ...

critical values : $x=0$, $x=-2$, $x=2$, $x=-1$



we want where $y > 0$

so... $\boxed{\{x \in \mathbb{R} : x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}}$

Question 3 continued

(Total for Question 3 is 6 marks)

4.

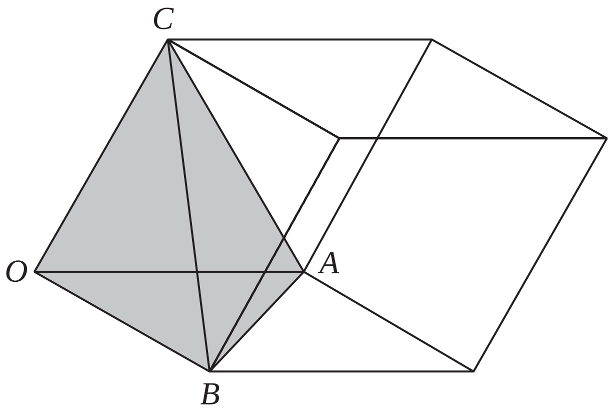


Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$, where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron. **(4)**

(b) Find the volume of glass contained in this parallelepiped. **(5)**

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture. **(1)**

a) $\overrightarrow{OC} = \begin{pmatrix}1\\1\\2\end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix}0\\3\\1\end{pmatrix}$ $\Big\}$ $\overrightarrow{OC} \times \overrightarrow{OB} = \begin{pmatrix}1\\1\\2\end{pmatrix} \times \begin{pmatrix}0\\3\\1\end{pmatrix}$

* area OBC = area ABC

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix}1 & 2\\3 & 1\end{vmatrix} - j\begin{vmatrix}1 & 2\\0 & 1\end{vmatrix} + k\begin{vmatrix}1 & 1\\0 & 3\end{vmatrix}$$

$$= \begin{pmatrix}-5\\-1\\3\end{pmatrix}$$

$$|\overrightarrow{OC} \times \overrightarrow{OB}| = \sqrt{(-5)^2 + (-1)^2 + 3^2}$$

$$= \sqrt{35}$$

so area of glued face $= \frac{1}{2} \times \sqrt{35}$

$$= \boxed{\frac{\sqrt{35}}{2}} \text{ m}^2$$

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Question 4 continued

b) $\overrightarrow{OA} \times \overrightarrow{OB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -2 \\ 6 \end{pmatrix}$

Area of tetrahedron : $\frac{1}{6} \left| \overrightarrow{OC} \cdot (\overrightarrow{OA} \times \overrightarrow{OB}) \right| = \frac{1}{6} \left| \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -2 \\ 6 \end{pmatrix} \right|$

$= \frac{1}{6} \left| -2 + 12 \right|$

$= \frac{10}{6}$

Volume of parallelepiped : $6 \times \frac{10}{6} = 10$

(since 6 tetrahedrons fit in a parallelepiped)

so volume of glass : $10 - \frac{10}{6} = \boxed{\frac{25}{3}}$

c) Inaccuracy of measurements of sides or concrete may not be uniformly smooth on the surface.

(Total for Question 4 is 10 marks)

5.

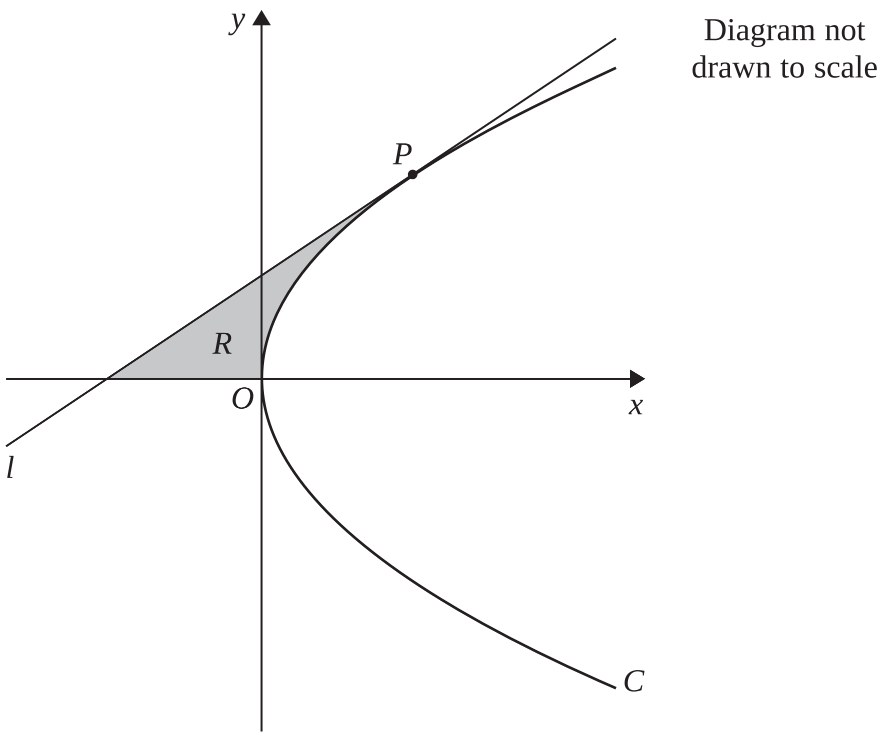


Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\dfrac{dy}{dx} = \dfrac{2a}{y}$]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\dfrac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

a) $y^2 = 16(4p)^2$
$= 64p^2$
$y = 8p$ hence P is a general point on C.

Question 5 continued

b) $\frac{dy}{dx} = \frac{2a}{8p}$

$= \frac{a}{4p}$

$\therefore\ y - 8p = \frac{a}{4p}(x - 4p^2)$

$y = \frac{a}{4p}x - ap + 8p$

$4a = 16 \rightarrow a = 4$

so $y = \frac{4}{4p}x - 4p + 8p$

$y = \frac{1}{p}x + 4p$

$\times p$

$py = x + 4p^2$

c) directrix : $x = -4$

$x = -4,\ y = \frac{10}{3}\ :\ p\left(\frac{10}{3}\right) = -4 + 4p^2$

$4p^2 - \frac{10}{3}p - 4 = 0$

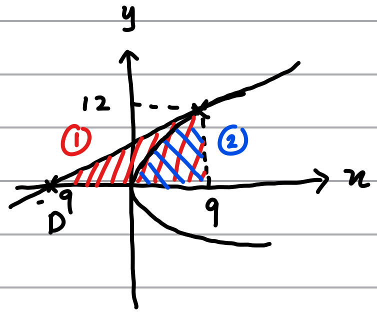
$\left.\begin{array}{l} a = 4 \\ b = \frac{-10}{3} \\ c = -4 \end{array}\right\}$ by quadratic formula : $p = \frac{3}{2},\ p = -\frac{2}{3}$

$p > 0\ \therefore\ p = \frac{3}{2}$ only

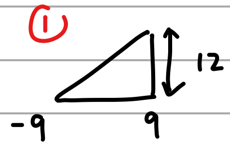
so $\frac{3y}{2} = x + 9$

$y = 0 : x = -9\quad \therefore\ D(-9, 0)$

and $P(9, 12)$



R = 1



$- \int_0^9 [4x^{\frac{1}{2}}]\,dx$ 2

$= \frac{1}{2}(18)(12) - \left[\frac{8}{3}x^{\frac{3}{2}}\right]_0^9$

$= 108 - 72$

$= \boxed{36}$

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

TOTAL FOR PAPER IS 40 MARKS